Title: Image Compression Activity (Digital Pictionary)

Brief Overview:

A critical component of multimedia communications is the ability to transport real-time and non-real time video and images over a network. A major issue is that video traffic is very bandwidth intensive, often making it prohibitive to carry video effectively in its original uncompressed format.

In this exercise, students will deduce algorithms for compressing the information contained within the pixels of various sample "images," in an attempt to reduce the volume of information that needs to be transmitted across the network. This is done by members of the team agreeing on an encoding scheme for the pixels within each image.

Each team is split into two equal groups, with one half acting as the senders or "encoders" and the other half acting as the receivers or "decoders". The groups are separated such that the image is initially seen only by the senders. The senders encode the pixels of the image onto index cards which are passed to the receivers. The receivers try to decode the pixels and reconstruct the image as quickly and as accurately as possible.

Links to NCTM Standards:

- **Mathematics as Problem Solving:**
  In a competitive setting, students will match wits against other groups within the class to optimize the compression scheme that is most efficient. There is no single correct answer, and the effectiveness of the scheme depends on its simplicity, as well as on the team's ability to implement it without errors.

- **Mathematics as Communication:**
  This exercise relies on effective communication between the team members to implement the algorithm for compressing the images. If the algorithm is not clearly understood by both groups within the team (i.e., senders and receivers), it is impossible to reconstruct the original image.

- **Mathematics as Reasoning:**
  Students will need to utilize their reasoning skills to anticipate an encoding scheme that will be effective for subsequent images. Although they have not yet seen the image to which this algorithm will be applied, students can generalize their experiences from the effectiveness of previous images to arrive at the next algorithm.

- **Mathematical Connections:**
  By analyzing the effectiveness of their algorithms in encoding and decoding the series of images, the students begin to deduce the requirements for a more generic compression algorithm that would effectively handle any "typical" image. Although the generic algorithm may not be optimal for a given image, it will provide good performance across the board for a variety of images. Video compression standards such as JPEG and MPEG work on this principle of overall performance rather than optimization.

- **Number Systems and Number Theory:**
  The encoding schemes used by the students are based on based on Binary (Base 2) and Hexadecimal (Base 16). The binary number system represents the language of digital electronics, and hexadecimal may be used by some students as a more efficient way of representing binary numbers. Through this exercise students become much more adept at using binary numbers.

Grade/Level:

Students in grades 9-11

Duration/Length:

One hour

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Binary number system
- Images as a collection of pixels, with "x" bits needed to represent each pixel based on number of colors supported

Objectives:

Students will:

- apply their understanding of binary numbers to encode and decode a series of images.
- work in a team to arrive at effective algorithms for compressing the images.
- compete with other teams within the class to determine which algorithms are most effective, where effectiveness is determined by both speed and accuracy.
- relate the lessons of this exercise to previous discussions on video encoding and transmission, allowing them to generalize the concepts within this exercise to broader applications.

Materials/Resources/Printed Materials:

- Blank grid sheets
- Images (patterns on the grid)
- Index cards

Development/Procedures:

The class is organized in the following manner:

1. The class is divided into teams of approximately four students each.
2. Each team is divided into two halves. The halves do not have to contain the same number of students. One half is designated the "senders" and the other half the "receivers".

The rules of the Pictionary Activity are as follows:

1. Once the game begins, the two halves of the team may only communicate using index cards with binary or hexadecimal digits. The index cards may also have colons, commas, and dashes. Index cards may be left blank. No pictures (lines, squares, etc.) may be drawn on the index cards.
2. During the game, the members on each half of the team may quietly whisper to each other to help facilitate writing down information on the index cards or reading information from the cards.
3. The first team to have the receivers correctly recreate the image on the blank graph paper wins the game.

The logistics of the activity are as follows:

1. Each team has three desks in a line spaced about 5 feet apart. Each half of the team occupies one of the end desks.
2. The middle desk is placed in between the two halves of the team. The index cards are passed by placing them on and picking them up from this middle desk. This allows multiple cards to be lined up waiting on the middle desk. It also prevents team members from getting too close to the other half of the team and "accidentally" seeing the picture. Team members may not cross the boundary dictated by the middle desk.
3. The senders on each team are given the image face down. On the go signal, all senders flip over the image and begin converting the image to numbers on the index cards.
4. The receivers may send index cards back to the senders. This allows for error correcting or for verifying a confusing index card.
5. In between each round the teams can discuss strategy and improve their algorithm.

Compression Algorithms

Following are compression algorithms that students will most likely use:
1. Send the coordinates of the squares which are filled in.
2. Send the coordinates of the endpoints of a line.
3. Send the coordinates of the corners of a filled in rectangle.
4. Send a "reverse" code which indicates that the defined squares are not filled in, and all other squares are filled in. This is useful for the patterns where there are more black squares than white squares.

Following are patterns that are difficult to efficiently compress:
1. Small squares.
2. Checkerboard patterns
3. Random filled in squares

Error Checking

Following might be reasons to send an index card from the receivers to the senders:
1. The receivers cannot read an index card due to messy handwriting.
2. The receivers cannot understand a sequence of numbers. Perhaps a set of coordinates is outside of the grid because of a mistake.

Other Notes:
1. Senders may want to number their index cards so that the receivers can recreate the order of the index cards later, if desired.

2. Receivers may need multiple blank grids in case they make a mistake.

Performance Assessment:

This section should describe how teachers can assess students' progress towards the objectives as they do the activities/tasks outlined in the rest of the learning unit. In other words, this section should be a guide to teachers on how to assess students' performance (it may include scoring rubrics, for example).

Pattern Recognition

For each new image, the student recognizes patterns of squares which can be easily represented in an algorithm.

For example in the first pattern, the number of black squares is small and their locations scattered randomly, so it is not necessary to communicate a group or sequence of black squares in one exchange. The L-shaped image, however, can be divided into two rectangles.

Pattern Encoding Algorithm

Once a pattern or patterns has been identified, the student needs to develop a way to represent the pattern efficiently in binary or hexadecimal numbers. The more efficient and robust, the better.

For the L-shaped image consisting of two rectangular patterns, an efficient method is to communicate rectangles by the four coordinates. For lines, an approach could be to send the starting coordinate and the length.

There are many possible ways of representing patterns in numbers. The better ways are both efficient (less numbers for more boxes) and also quick to decipher and not prone to error.

Extension/Follow Up:

List or describe possible extensions or follow-up activities for your unit. Don’t go into great detail unless absolutely necessary.

For further in-depth understanding of this exercise, it is good to discuss the various approaches (i.e., encoding algorithms) to a particular image and to identify the pros and cons of each approach. The approach generally differs from image to image, so it may be a good challenge to identify one approach that is applicable to many patterns and achieves the best overall efficiency. In other words, an approach may be inefficient in certain cases, but provides a good general solution to a typical image.

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